

# Entropy Bound of Horizons for Accelerating, Rotating and Charged Plebanski-Demianski Black Hole

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We first review the accelerating, rotating and charged Plebanski-Demianski (PD) black hole, which includes the Kerr-Newman rotating black hole and the Taub-NUT spacetime. The main feature of this black hole is that it has 4 horizons like event horizon, Cauchy horizon and two accelerating horizons. In the non-extremal case, the surface area, entropy, surface gravity, temperature, angular velocity, Komar energy and irreducible mass on the event horizon and Cauchy horizon are presented for PD black hole. The entropy product, temperature product, Komar energy product and irreducible mass product are found for event horizon and Cauchy horizon. Also their sums are also found for both horizons. All these relations are found to be depend on mass of the PD black hole and other parameters. So all the products are not universal for PD black hole. The entropy and area bounds for two horizons are investigated. Also we found the Christodoulou-Ruffini mass for extremal PD black hole. Finally, using first law of thermodynamics, we also found the Smarr relation for PD black hole.

Keywords: Thermodynamics, Entropy, Black hole horizons.

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## I. INTRODUCTION

In 1981, Bekenstein [1] proposed the universal bound on the entropy of a macroscopic object of maximal radius  $R$  bearing energy  $E$  in the form  $S \leq \frac{2\pi ER}{\hbar}$ . But the derivation of the entropy bound was criticized by Unruh, Wald and Pelath [2–4]. Bekenstein refused their criticism and showed that buoyancy is so negligible such that it does not spoil the entropy bound derivation [5, 6]. In various occasions, this type of bound has been found in some literatures by the same author [7–9]. In 1992, Zaslavskii [10] modified the entropy bound by incorporating the charge of a black hole. After that Bekenstein and Mayo [11] and Hod [12, 13] have obtained an upper entropy bound for a charged Reissner-Nordstrom black hole, in the form  $S \leq \frac{2\pi R}{\hbar} \left( E^2 - \frac{e^2}{2R} \right)$ , where  $e$  is the electric charge of the black hole. Shimomura et al [14] have also obtained the entropy bound for charged black hole. Including the angular momentum, Linet [15, 16] and Qiu et al [17] obtained the upper bound of the entropy on the more general Kerr-Newman black hole. Wand and Abdalla [18] studied the entropy bound for a spinning object falling into anti de Sitter (AdS) black holes including  $(3 + 1)$ -dimensional Kerr-AdS black holes and  $(2 + 1)$ -dimensional Banados-Teitelboim-Zanelli (BTZ) black holes. Jing [19] obtained the Cardy-Verline formula and the entropy bounds in Kerr-Newman-AdS<sub>4</sub>/dS<sub>4</sub> black hole.

The product of horizon areas or the entropy product of horizons of black hole are very important in the study of black hole physics. The two horizons of the black hole are namely inner/Cauchy horizon ( $\mathcal{H}^-$ ) and outer/event horizon ( $\mathcal{H}^+$ ). Now it is known that the Cauchy horizon ( $\mathcal{H}^-$ ) is an infinite blue-shift surface, but the event horizon ( $\mathcal{H}^+$ ) is an infinite red-shift surface [20]. For stationary axially symmetric black holes, the entropy product of horizons are often independent of the mass of the black hole [21–25]. Such products depend on the charge and angular momentum of the black hole. In some cases this relation may be depend on the mass of the black hole [26–28]. So the entropy sum and other thermodynamic relations have been studied [29–33]. In some cases these relations may be independent of black hole mass and some cases these depend on black hole mass. For regular axisymmetric and stationary spacetime of an Einstein-Maxwell system with surrounding matter has a regular Cauchy horizon ( $\mathcal{H}^-$ ), which always occur inside the event horizon ( $\mathcal{H}^+$ ) if and only if the angular momentum  $J$  and charge  $Q$  of the black hole do not vanish simultaneously. In this case, the Cauchy horizon ( $\mathcal{H}^-$ ) becomes singular and tends to a curvature singularity when  $J$  and  $Q$  tend to zero [34–36]. In Boyer-Lindquist coordinates, the existence of Cauchy horizon describes that the stationary and axisymmetric Einstein-Maxwell electro-vacuum equations are hyperbolic in the interior vicinity of the event horizon ( $\mathcal{H}^+$ ). The two horizons  $\mathcal{H}^+$  and  $\mathcal{H}^-$  describe the future and past boundary of this hyperbolic region. If the Cauchy horizon exists i.e., if  $J$  and  $Q$  do not vanish simultaneously, then the product of the entropy (area) of the horizons  $\mathcal{H}^\pm$  for the Kerr-Newman black hole is independent of the mass of the black hole, but depends on the angular momentum

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$J$  and the charge  $Q$  explicitly [29].

Based on the entropy product and entropy sum, very recently, Xu et al [37] have obtained entropy (area) bound for event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) for Kerr, Kerr-Newman, Kerr-Newman in Gauss Bonnet gravity and Kerr-Taub-NUT black holes. They have actually taken the Penrose-like inequality for the upper area bound of event horizon ( $\mathcal{H}^+$ ). They also found that the electric charge  $Q$  diminishes the physical bound of entropy (area) for event horizon ( $\mathcal{H}^+$ ), while it enlarges that for Cauchy horizon ( $\mathcal{H}^-$ ); the angular momentum  $J$  enlarges them for Cauchy horizon ( $\mathcal{H}^-$ ), while it does nothing with that for event horizon ( $\mathcal{H}^+$ ); the NUT charge  $n$  always enlarges them for both event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ). With the ideas of their work, we now formulate the entropy bounds on event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) for more general accelerating, rotating and charged Plebanski-Demianski black hole. We also determine the entropy product, entropy sum, sum of the angular velocities, temperature-entropy relations, black hole mass and the entropy bounds for event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ). Finally we discuss the main results of the work.

## II. PLEBANSKI-DEMIANSKI BLACK HOLE

A large family of Einstein-Maxwell electro-vacuum solutions of algebraic type  $D$  was presented by Plebanski and Demianski [38]. This includes the Kerr-Newman rotating black holes, the Taub-NUT spacetime, the (anti-) de Sitter metric and also their arbitrary combination. The resulting black holes may accelerate due to conical singularities (such as cosmic strings). The general form of the metric contains eight free parameters which characterize the mass, charge (electric and magnetic), angular momentum, rotation parameter, the NUT parameter, acceleration of the sources and the cosmological constant [39–43]. The new form of accelerating, rotating and charged Plebanski-Demianski (PD) black hole metric is given by [39]

$$ds^2 = \frac{1}{\Omega^2} \left[ -\frac{Q}{\rho^2} \left\{ dt - \left( a \sin^2\theta + 4l \sin^2\frac{\theta}{2} \right) d\phi \right\}^2 + \frac{\rho^2}{Q} dr^2 + \frac{P}{\rho^2} \left\{ a dt - (r^2 + (a+l)^2) d\phi \right\}^2 + \frac{\rho^2}{P} \sin^2\theta d\theta^2 \right] \quad (1)$$

where  $\Omega$ ,  $\rho^2$ ,  $Q$  and  $P$  are in the following:

$$\Omega = 1 - \frac{\alpha}{\omega} (l + a \cos\theta) r, \quad (2)$$

$$\rho^2 = r^2 + (l + a \cos\theta)^2, \quad (3)$$

$$Q = \left[ (\omega^2 k + e^2 + g^2) \left( 1 + \frac{2\alpha l}{\omega} r \right) - 2Mr + \frac{\omega^2 k}{a^2 - l^2} r^2 \right]$$

$$\times \left[ 1 - \frac{\alpha(a+l)}{\omega} r \right] \left[ 1 + \frac{\alpha(a-l)}{\omega} r \right], \quad (4)$$

$$P = (1 - a_3 \cos\theta - a_4 \cos^2\theta) \sin^2\theta, \quad (5)$$

$$a_3 = \frac{2\alpha a}{\omega} M - \frac{4\alpha^2 a l}{\omega^2} (\omega^2 k + e^2 + g^2), \quad (6)$$

$$a_4 = -\frac{\alpha^2 a^2}{\omega^2} (\omega^2 k + e^2 + g^2), \quad (7)$$

$$\left[ \frac{\omega^2}{a^2 - l^2} + 3\alpha^2 l^2 \right] k = 1 + \frac{2\alpha l}{\omega} M - \frac{3\alpha^2 l^2}{\omega^2} (e^2 + g^2) \quad (8)$$

Here,  $e$  is the electric charge,  $g$  is the magnetic charge,  $a$  is the angular momentum ( $= J/M$ ),  $l$  is the NUT parameter,  $\alpha$  is the acceleration and  $\omega$  is the rotation parameter. The PD black hole metric reduces to the following well known black hole metrics as special cases: (i) Kerr-Newman-Taub-NUT ( $\alpha = g = 0$ ) [44, 45] (ii) Kerr-Taub-NUT ( $\alpha = e = g = 0$ ) [46] (iii) Taub-NUT ( $\alpha = a = e = g = 0$ ) [47] (iv) Kerr-Newman ( $\alpha = l = g = 0$ ) [48] (v) Kerr ( $\alpha = l = e = g = 0$ ) [49] (vi) Riessner-Nordstrom ( $\alpha = a = l = g = 0$ ) (vii) Schwarzschild ( $\alpha = a = l = e = g = 0$ ) and (viii) C-metric ( $a = l = 0$ ) [50].

Horizons of the black hole found when  $\frac{1}{g_{rr}} = 0$ , i.e.,  $Q = 0$ . So the horizons are located at

$$r_{\pm} = \frac{a^2 - l^2}{\omega^2 k} \left[ \left( M - \frac{\alpha l}{\omega} (\omega^2 k + e^2 + g^2) \right) \pm \sqrt{\left( M - \frac{\alpha l}{\omega} (\omega^2 k + e^2 + g^2) \right)^2 - \frac{\omega^2 k}{a^2 - l^2} (\omega^2 k + e^2 + g^2)} \right] \quad (9)$$

with  $|a| > |l|$ . Here  $r_+$  represents the outer/event horizon ( $\mathcal{H}^+$ ) and  $r_-$  represents the inner/Cauchy horizon ( $\mathcal{H}^-$ ). Also  $r = \pm \frac{\omega}{\alpha(a \pm l)}$  are known as the accelerating horizons. While  $g_{tt} = 0$ , i.e.,  $Q = a^2 P$  describes the ergosphere. Here we are interested to study only the event horizon  $\mathcal{H}^+$  and Cauchy horizon  $\mathcal{H}^-$ . Now define the product and addition of the roots  $r_+$  and  $r_-$  by  $A = r_+ r_-$  and  $B = r_+ + r_-$ , where

$$A = \frac{(a^2 - l^2)}{\omega^2 k} (\omega^2 k + e^2 + g^2) \quad (10)$$

and

$$B = \frac{2(a^2 - l^2)}{\omega^2 k} \left( M - \frac{\alpha l}{\omega} (\omega^2 k + e^2 + g^2) \right) \quad (11)$$

In order to avoid naked singularity,  $r_{\pm}$  must be real. So we have the restriction:

$$\left( M - \frac{\alpha l}{\omega} (\omega^2 k + e^2 + g^2) \right)^2 \geq \frac{\omega^2 k}{a^2 - l^2} (\omega^2 k + e^2 + g^2) \quad (12)$$

with

$$k \geq 0 \quad \text{and} \quad M \geq \frac{\alpha l}{\omega} (\omega^2 k + e^2 + g^2) \quad (13)$$

If we take the equality of the expression (12), we must have  $r_+ = r_-$ . In this situation, the two horizons will coincide and the black hole will be called the *extremal black hole*. Now we assume that the black hole is non-extremal (i.e., if there exists a trapped surface interior of the outer horizon) so  $r_+ > r_-$ . Now the surface area of the PD black hole on  $\mathcal{H}^\pm$  is obtained by [29]

$$\mathcal{A}_\pm = \int \int \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi = \frac{4\pi\omega^2(r_\pm^2 + (a+l)^2)}{(\omega - l\alpha r_\pm)^2 - a^2\alpha^2 r_\pm^2} \quad (14)$$

So the entropies on the horizons  $\mathcal{H}^\pm$  are

$$S_\pm = \frac{\mathcal{A}_\pm}{4} = \frac{\pi\omega^2(r_\pm^2 + (a+l)^2)}{(\omega - l\alpha r_\pm)^2 - a^2\alpha^2 r_\pm^2} \quad (15)$$

Now the surface gravities on the horizons  $\mathcal{H}^\pm$  can be obtained by

$$\kappa_\pm = \frac{r_\pm - r_\mp}{2(r_\pm^2 + (a+l)^2)} \quad (16)$$

Since Chen et al [24] defined  $T_- = -T_+|_{r_+ \leftrightarrow r_-}$  so the temperatures on the horizons  $\mathcal{H}^\pm$  are given by

$$T_\pm = \frac{|\kappa_\pm|}{2\pi} = \pm \frac{r_\pm - r_\mp}{4\pi(r_\pm^2 + (a+l)^2)} \quad (17)$$

Since  $r_+ > r_-$ , so  $T_+ < T_-$ . Hence we may say that the Cauchy horizon is hotter than the event horizon. The product of the temperatures on the both horizons  $\mathcal{H}^\pm$  yields to be

$$T_+ T_- = \frac{B^2 - 4A}{16\pi^2 [A^2 + (a+l)^2(B^2 - 2A) + (a+l)^4]} \quad (18)$$

Curir [51] introduced the ‘area sum’ and ‘entropy sum’ of Kerr BH for the interpretation of the spin entropy of the area of the inner horizon. Now for PD black hole the product, addition and subtraction of two horizons ( $\mathcal{H}^\pm$ ) entropies are obtained as in the following:

$$S_+ S_- = \frac{F}{G}, \quad (19)$$

$$S_+ + S_- = \frac{H}{G}, \quad (20)$$

and

$$S_+ - S_- = \frac{I}{G} \quad (21)$$

where

$$F = \pi^2 \omega^4 [A^2 + (a+l)^4 + (a+l)^2(B^2 - 2A)] , \quad (22)$$

$$G = (a^2 - l^2)^2 \alpha^4 A^2 + 2l\alpha^3 \omega (a^2 - l^2) AB$$

$$- \omega^2 \alpha^2 (a^2 - l^2) (B^2 - 2A) + 4l^2 \alpha^2 \omega^2 A - 2l\alpha \omega^3 B + \omega^4 , \quad (23)$$

$$H = \pi\omega^2 [2\alpha^2(l^2 - a^2)A^2 - 2l\alpha\omega AB$$

$$+ \{(a+l)^2(l^2 - a^2)\alpha^2 + \omega^2\}(B^2 - 2A)$$

$$- 2l\alpha\omega(a+l)^2 B + 2\omega^2(a+l)^2] , \quad (24)$$

$$I = \pi\omega^2(r_+ - r_-) [\{\omega^2 + (a^2 - l^2)(a+l)^2\alpha^2\}(r_+ - r_-)$$

$$+ 2l\alpha\omega\{(a+l)^2 - A\}] . \quad (25)$$

From the above relations we may obtain the sum of entropy inverse

$$\frac{1}{S_+} + \frac{1}{S_-} = \frac{H}{F} \quad (26)$$

The angular velocities on the horizons  $\mathcal{H}^\pm$  are obtained by

$$\Omega_\pm = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a}{r_\pm^2 + (a+l)^2} \quad (27)$$

The sum of the angular velocities on  $\mathcal{H}^\pm$  are

$$\Omega_+ + \Omega_- = \frac{a[B^2 - 2A + 2(a+l)^2]}{A^2 + (a+l)^2(B^2 - 2A) + (a+l)^4} \quad (28)$$

The Komar [52] energy for  $\mathcal{H}^\pm$  is given by

$$E_\pm = 2T_\pm S_\pm = \pm \frac{\omega^2}{2} \frac{r_\pm - r_\mp}{(\omega - l\alpha r_\pm)^2 - a^2\alpha^2 r_\pm^2} \quad (29)$$

The product of Komar energies on the two horizons ( $\mathcal{H}^\pm$ ) are obtained by

$$E_+ E_- = \frac{\omega^4}{4G} (B^2 - 4A) \quad (30)$$

Penrose et al [53] shown that when a black hole (eg. Kerr black hole) is undergoing any transformations, the surface area of the horizon increases, which is known as Penrose process. Independently, Christodoulou [54] also shown that the mass of the black hole unchanged by any process, which is known as Christodoulou’s “irreducible mass” denoted by  $\mathcal{M}$ . But most of the processes, it was seen that  $\mathcal{M}$  increases and during reversible process this quantity also does not change. From this result, it was conclude that there exists a relation between the area and irreducible mass. The irreducible mass  $\mathcal{M}_\pm$  for the horizons  $\mathcal{H}^\pm$  can be defined by [29]

$$\mathcal{M}_\pm = \sqrt{\frac{\mathcal{A}_\pm}{16\pi}} \quad (31)$$

Now the product of the irreducible mass for PD black hole is obtained by

$$\mathcal{M}_+ \mathcal{M}_- = \frac{1}{4\pi} \sqrt{\frac{F}{G}} \quad (32)$$

From all the products of the thermodynamic quantities for  $\mathcal{H}^\pm$ , we observe that these products completely depend on the mass of PD black hole  $M$  and other parameters. So we may conclude that the products are not universal, while Pradhan [30] claimed that for charged rotating Kerr-Newman black hole, the thermodynamic quantities are not also universal except the area product and entropy product (because they do not depend of black hole mass). The relation of temperatures and entropies on the two horizons  $\mathcal{H}^\pm$  are [51]

$$T_+S_+ - T_-S_- = \frac{\omega^2}{4G} (r_+ - r_-)[(a^2 - l^2)\alpha^2 B + 2l\alpha\omega] \quad (33)$$

If there is no acceleration of the black hole i.e.,  $\alpha = 0$ , we obtain  $T_+S_+ = T_-S_-$ . This reads that the central charge for two-horizons black hole are same. But for accelerating black hole,  $T_+S_+ \neq T_-S_-$ . Since  $r_+ > r_-$ , so we have  $\mathcal{A}_+ > \mathcal{A}_- \geq 0$ . Now from the entropy product, we find [37]

$$S_+ \geq \sqrt{S_+S_-} = \sqrt{\frac{F}{G}} \geq S_- \geq 0. \quad (34)$$

Also from the entropy sum [37], we obtain

$$\frac{H}{G} = S_+ + S_- \geq S_+ \geq \frac{S_+ + S_-}{2} = \frac{H}{2G} \geq S_- \quad (35)$$

Thus the entropy bounds for event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) of PD black hole are

$$S_+ \in \left[ \frac{H}{2G}, \frac{H}{G} \right], \quad S_- \in \left[ 0, \sqrt{\frac{F}{G}} \right] \quad (36)$$

So the area bounds for event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) of PD black hole are

$$\mathcal{A}_+ \in \left[ \frac{2H}{G}, \frac{4H}{G} \right], \quad \mathcal{A}_- \in \left[ 0, 4\sqrt{\frac{F}{G}} \right] \quad (37)$$

and hence the bounds of the irreducible mass for event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) of PD black hole are obtained as

$$\mathcal{M}_+ \in \left[ \sqrt{\frac{H}{8\pi G}}, \sqrt{\frac{H}{4\pi G}} \right], \quad \mathcal{M}_- \in \left[ 0, \frac{1}{\sqrt{4\pi}} \left( \frac{F}{G} \right)^{\frac{1}{4}} \right] \quad (38)$$

From the above bounds for horizons entropy, area and irreducible mass, we understand that the lower and upper bounds completely depend on all the parameters like PD black hole mass  $M$ , angular momentum  $a$ , electric charge  $e$ , magnetic charge  $g$ , rotation parameter  $\omega$ , NUT parameter  $l$  and the acceleration parameter  $\alpha$ . In particular, for extremal PD black hole,  $r_+ = r_-$ , we obtain  $\mathcal{A}_+ = \mathcal{A}_-$ ,  $S_+ = S_-$ ,  $\Omega_+ = \Omega_-$ ,  $\mathcal{M}_+ = \mathcal{M}_-$ ,  $\kappa_+ = \kappa_- = 0$ ,  $T_+ = T_- = 0$  and  $E_+ = E_- = 0$ . So surface gravity, temperature and Komar energy vanish on the horizon of the PD extremal black hole. For extremal PD black hole, we may obtain the area  $A = \frac{H}{8G}$ , entropy  $S = \frac{H}{2G}$  and

angular velocity  $\Omega = \frac{a}{A+(a+l)^2}$  with the conditions  $I = 0$ ,  $B^2 = 4A$  and  $H^2 = 4FG$ . And hence the Christodoulou-Ruffini [55] mass for extremal PD black hole will be

$$\begin{aligned} \mathcal{M}_{CR} = & [l^3\alpha^3\omega(a^2 - l^2)((a^2 - l^2) - 2(e^2 + g^2)) \\ & + l\alpha\omega^3(3(a^2 - l^2) + 2(e^2 + g^2)) + \omega(3l^2\alpha^2(a^2 - l^2) + \omega^2) \\ & \times \sqrt{\omega^2(a^2 - l^2 + e^2 + g^2) - l^2\alpha^2(a^2 - l^2)(e^2 + g^2)}] \\ & \times [\omega^2 - l^2\alpha^2(a^2 - l^2)]^{-2} \end{aligned} \quad (39)$$

with the condition

$$\omega^2(a^2 - l^2 + e^2 + g^2) \geq l^2\alpha^2(a^2 - l^2)(e^2 + g^2). \quad (40)$$

In particular, for  $\alpha = g = 0$ , we can recover the Christodoulou-Ruffini mass for extremal Kerr-Newman-Taub-NUT black hole [29] i.e.,  $\mathcal{M}_{CR} = \sqrt{a^2 + e^2 - l^2}$ .

Finally, we demonstrate the first law of thermodynamics of event horizon ( $\mathcal{H}^+$ ) and Cauchy horizon ( $\mathcal{H}^-$ ) for PD black hole. The first law of thermodynamics are given by [37]

$$dM = \pm T_\pm dS_\pm + \Omega_\pm dJ + \mathcal{E}_\pm de + \mathcal{G}_\pm dg + \Phi_\pm dl \quad (41)$$

where  $\mathcal{E}_\pm$ ,  $\mathcal{G}_\pm$  and  $\Phi_\pm$  are the electromagnetic potentials for electric charge, magnetic charge and NUT charge respectively defined by [37]

$$\begin{aligned} \mathcal{E}_\pm &= \left( \frac{\partial M}{\partial e} \right)_{S_\pm, J, g, l}, \quad \mathcal{G}_\pm = \left( \frac{\partial M}{\partial g} \right)_{S_\pm, J, e, l}, \\ \Phi_\pm &= \left( \frac{\partial M}{\partial l} \right)_{S_\pm, J, e, g} \end{aligned} \quad (42)$$

Here  $M$  scales as  $[\text{length}]^1$ ,  $S_\pm$  scales as  $[\text{length}]^2$ ,  $J$  scales as  $[\text{length}]^2$ ,  $e$  scales as  $[\text{length}]^1$ ,  $g$  scales as  $[\text{length}]^1$  and  $l$  scales as  $[\text{length}]^1$  [37]. So we may obtain

$$M = 2(\pm T_\pm S_\pm + \Omega_\pm J) + \mathcal{E}_\pm e + \mathcal{G}_\pm g + \Phi_\pm l \quad (43)$$

which behave as the Smarr relation [56, 57] of PD black hole for event and Cauchy horizons  $\mathcal{H}^\pm$ .

### III. DISCUSSIONS

In this work, first we have reviewed the most general black hole i.e., the accelerating, rotating and charged Plebanski-Demianski (PD) black hole, which includes the Kerr-Newman rotating black hole and the Taub-NUT spacetime. Neglecting some parameters involved in the PD black hole metric, we can recover the Kerr-Newmann-Taub-NUT black hole, Kerr-Taub-NUT black hole, Taub-NUT black hole, Kerr-Newmann black hole, Kerr

black hole, Riessner-Nordstrom black hole, Schwarzschild black hole and C-metric. The main feature of this PD black hole is that it has 4 horizons like event horizon ( $\mathcal{H}^+$ ), Cauchy horizon ( $\mathcal{H}^-$ ) and two accelerating horizons which are located at  $r_+$ ,  $r_-$  and  $r = \pm \frac{\omega}{\alpha(a \pm l)}$  respectively. Since the event horizon ( $\mathcal{H}^+$ ) forms outside the Cauchy horizon ( $\mathcal{H}^-$ ), so in the non-extremal case (i.e.,  $r_+ > r_-$ ), the surface area, entropy, surface gravity, temperature, angular velocity, Komar energy and irreducible mass on the event horizon and Cauchy horizon are presented for PD black hole. Since  $r_+ > r_-$ , so  $T_+ < T_-$  i.e., we may say that the Cauchy horizon is hotter than the event horizon. But for extremal case (two horizons are identical i.e.,  $r_+ = r_-$ ), we have found that the surface gravity, temperature and Komar energy vanish on the horizon. The entropy product, temperature product, Komar energy product and irreducible mass product are found for event horizon and Cauchy horizon. Also their sums are also found for both horizons. All these relations are found to be depend on mass of the PD black hole and other parameters involved. So all the products are said to be not universal for PD black hole. Also from the relations of temperature and entropy of the horizons  $\mathcal{H}^\pm$ , we found

that  $T_+S_+ \neq T_-S_-$ . If there is no acceleration of the black hole i.e.,  $\alpha = 0$ , we can recover  $T_+S_+ = T_-S_-$ . This reads that for non-accelerating black hole, the central charge for two-horizons black hole are same. Since  $r_+ > r_-$ , so we have  $\mathcal{A}_+ > \mathcal{A}_- \geq 0$  and from the entropy product and sum, we found the bounds of the entropy for  $\mathcal{H}^\pm$  as  $S_+ \in [\frac{H}{2G}, \frac{H}{G}]$ ,  $S_- \in [0, \sqrt{\frac{F}{G}}]$ . Also we found the bounds of area for  $\mathcal{H}^\pm$ . All these bounds are completely depend on the 7 parameters  $M, e, g, \omega, \alpha, a, l$ . For extremal (two horizons are identical) black hole, we found the area  $A = \frac{H}{8G}$ , entropy  $S = \frac{H}{2G}$  and angular velocity  $\Omega = \frac{a}{A+(a+l)^2}$  with the conditions  $I = 0$ ,  $B^2 = 4A$  and  $H^2 = 4FG$ . Also we found the Christodoulou-Ruffini mass for extremal PD black hole. Finally, using first law of thermodynamics, we also found the Smarr relation for PD black hole.

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